CISC124 – Today's Topics

- Quiz 2
- Numeric representations
- Internal representation of numbers
  - Integers (two's complement)
  - Real numbers (IEEE 754 floating point)

Winter 2019 F. de la Parra 1

•1

Quiz 2

Location and Hours:

- Quiz 2 will be written in Jeffery 155, at the beginning of your lab session
- Mon Feb 25, 9:30 am – 10:30 am
- Mon Feb 25, 2:30 pm – 3:30 pm
- Tue Feb 26, 8:30 am – 9:30 am
- Wed Feb 27, 2:30 pm – 3:30 pm

Format:

- One coding question
- A set of multiple choice questions

Winter 2019 F. de la Parra 2

•2

Quiz 2

Topics (Everything covered from Jan 29 until Feb 13) :

- 2D arrays
- Ragged arrays
- Aliasing. System properties
- Exception throwing and handling. Exception classes
- Try-catch-finally statement
- Wrapper classes
- Foundational classes: Math, String, StringTokenizer
- Method overloading
- File IO: text and binary files
- Software qualities
- Classes, objects, encapsulation

Winter 2019 F. de la Parra 3

•3

Numeric representations

- Representation of a numeric value (counting or measuring intuition) using a system of digits and a method to combine those digits using powers of a base number
- Digits (coefficients) & Base (radix)
  - Decimal System → 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (Base r = 10)
  - Hexadecimal System → 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F (Base r = 16)
  - Octal System → 0, 1, 2, 3, 4, 5, 6, 7 (Base r = 8)
  - Binary → 0, 1 (Base r = 2)
- Method (Represent n-digit number by a polynomial of degree (n-1))
- (Integer) $N = d_{n-1} \times r^{n-1} + \ldots + d_0 \times r^0$
- (decimal) $D = d_{n-1} \times r^{n-1} + \ldots + d_0 \times r^0 + d_{-1} \times r^{-1} + \ldots + d_{-m} \times r^{-m}$

Winter 2019 F. de la Parra 4

•4

Numeric representations

- Examples:
- 1867 is a base 10 representation of the count 1867
  - Base 10 → $1867 = 1 \times 10^3 + 8 \times 10^2 + 6 \times 10^1 + 7 \times 10^0$
  - Base 16 → $74B = 7 \times 16^2 + 4 \times 16^1 + B \times 16^0$ (B is 11 decimal)
  - Base 8 → $5513 = 5 \times 8^3 + 5 \times 8^2 + 1 \times 8^1 + 3 \times 8^0$
  - Base 2 → $11101001011 = 1 \times 2^{10} + 1 \times 2^9 + 1 \times 2^8 + 0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 10 \times 2^1 + 1 \times 2^0$
- 1867.25 in base 10 :

$$1 \times 10^3 + 8 \times 10^2 + 6 \times 10^1 + 7 \times 10^0 + 2 \times 10^{-1} + 5 \times 10^{-2}$$

Winter 2019 F. de la Parra 5

•5

Numeric conversions

- Any other number representation to decimal → just replace the digits and radix by their decimal representations and do decimal math.
- Decimal number representation to others (hexadecimal, octal, binary) requires simple algorithm:
  1. Divide decimal number by new base (integer division) → remainder is last digit of new number representation
  2. Divide quotient of previous division by new base → remainder is second last digit of new number representation
  3. Perform steps 1 an 2 until quotient is 0 → remainder is left-most digit of new number representation
- Between number representations whose bases are powers of 2 (hexadecimal (16), octal (8), binary (2)) just complete a 1 or 2 step algorithm:
  - Convert each digit to its binary representation
  - Regroup them in groups of the following sizes: 4 for hexadecimal, 3 for octal, 1 for binary

Winter 2019 F. de la Parra 6

•6

Internal representation

- Internal representation of integers:
  - Two's complement encoding → positive and negative integers
  - Fixed number of bits to represent an integer: 8, 16, 32, 64
  - 1 sign (left-most) bit → 0 if positive integer, 1 if negative integer
  - Representation of positive integers → 0 sign bit plus unsigned binary representation of integer
  - Representation of negative integers → one is added to inverted representation of positive integer (0's are flipped to 1's and 0's to 1's)
  - Addition and subtraction are reduced to just unsigned binary addition. Easy to convert results back to decimal representation

Winter 2019 F. de la Parra 7

•7

Internal representation

- Example: integers in an 8-bit representation
  - Representation range → [-128, 127]
  - Positive integer 45 → 00101101
  - Negative integer -24 → 00011000 → 11100111 + 1 = 11101000
  - 45 – 24

```
00101101 +
11101000
--------------
```

Subtraction of integers in two's complement representation becomes just the addition of unsigned binary numbers

00010101 (two's complement) → 21 (decimal)

Winter 2019 F. de la Parra 8

•8

Internal representation

Overflow problem: arithmetic operations can produce results outside the representation range

- Positive integer 105 → 00101101
- Positive integer 34 → 00100010
- 105 + 34

```
00101101 +
00100010
--------------
01001111
```

→ 79 (wrong!?)

Winter 2019 F. de la Parra 9

•9

Internal representation

Internal representation of real numbers:

- Java follows the IEEE 754 Standard → float (32 bits, single precision), double (64 bits, double precision)
- Storage format for a java float consists of:
  - A sign bit → 0 - positive number, 1 - negative number
  - An exponent field → 8 bits, values 1-254 interpreted as *e*-127. 0 and 255 special.
  - A mantissa field → 23 bits that are the coefficients of a polynomial on r = 2

0 00000000 00000000000000000000000

sign *s* (1 bit) | exponent *e* (8 bits) | Mantissa (23 bits)

Value = $(-1)^s \times 2^{e-127} \times (1 + \sum_{i=1}^{23} b_{(23-i)} \times 2^i)$

Winter 2019 F. de la Parra 10

•10

Internal representation

Example:

b_{31} b_{30} b_{24} b_{23} b_0

0 01111100 01000000000000000000000

Value = $(-1)^0 \times 2^{124-127} \times (1 + 2^{-2})$

Value = $1 \times 2^{-3} \times (1.25)$

Value = 0.15625

Winter 2019 F. de la Parra 11

•11

Internal representation

Internal representation boundaries for a float:

Max. Value = $(-1)^0 \times 2^{254-127} \times (1 + \sum_{i=1}^{23} 2^i)$

is constant Float.MAX_VALUE = 3.4028235E38

Smallest Value = $(-1)^0 \times 2^{1-127} \times (1)$

is constant Float.MIN_NORMAL = 1.17549435E-38

Special values:
- 0 = $(-1)^0 \times 2^0 \times (1)$
- +Infinity = $(-1)^0 \times 2^{255} \times (1)$
- -Infinity = $(-1)^1 \times 2^{255} \times (1)$
- NaN = $(-1)^0 \times 2^{255} \times (1 + \sum_{i=1}^{23} 2^i)$

Winter 2019 F. de la Parra 12

•12