CISC-204*
Test #1
January 30, 2009

Student Number (Required) ______________________

Name or Qlink ID (Optional)________________________________

This is a closed book test. You may not refer to any resources other than the information sheet stapled to the back of the test. You may remove the information sheet.

This is a 50 minute test.

Please write your answers in ink. Pencil answers will be marked, but will not be reconsidered after the test papers have been returned.

The test will be marked out of 50.

Question 1	/15
Question 2	/15
Question 3	/10
Question 4	/10
TOTAL	/50

QUESTION 1. 15 Marks

(a) (10 marks) Using the rules of natural deduction, prove that

$$\neg (a \rightarrow b) \vdash a \land \neg b$$

Question 1 continued:

(b) (5 marks) Using the rules of natural deduction and the result of part **(a)**, prove that

$$\vdash \mathbf{(p \rightarrow q) \lor (q \rightarrow r)}$$

[Hint: start by applying the LEM to $(p \rightarrow q)$]

QUESTION 2: 15 Marks

Consider the Well-Formed formula

$$(((p \lor q) \rightarrow r) \land ((r \rightarrow (\neg q)) \land ((\neg q) \rightarrow (\neg p))))$$

(a) **(5 marks)** Draw the parse tree for this WFF

(b) **(10 marks)** Determine whether or not this formula can ever be true. Use any method in propositional logic to demonstrate the correctness of your answer.

QUESTION 3: 10 Marks

(a) **(5 marks)** Suppose we were forbidden to use the $\rightarrow$ symbol. Explain how we could rewrite all formulas in propositional logic so that none of them contained that symbol (without inventing any new symbols, of course).

(b) **(5 marks)** Suppose we were also forbidden to use the $\vee$ symbol. Explain how we could rewrite all formulas in propositional logic so that none of them contained any $\rightarrow$ or $\vee$ symbols (without inventing any new symbols, of course).

QUESTION 4: 10 Marks

Suppose you constructed a truth table for a formula and discovered that the final column had **FALSE** in every row. How can you use this information to create a theorem in propositional logic?

Bonus Lewis Carroll Question: (0 Marks)

What conclusion can you draw from the following propositions?

(a) No interesting poems are unpopular among people of real taste.

(b) No modern poetry is free from affectation.

(c) All *your* poems are on the subject of soap-bubbles.

(d) No affected poetry is popular among people of real taste.

(e) No ancient poem is on the subject of soap-bubbles.

The basic rules of natural deduction:

	introduction	*elimination*
$\wedge$	$\dfrac{\phi \quad \psi}{\phi \wedge \psi}\ \wedge\mathrm{i}$	$\dfrac{\phi \wedge \psi}{\phi}\ \wedge\mathrm{e}_1 \qquad \dfrac{\phi \wedge \psi}{\psi}\ \wedge\mathrm{e}_2$
$\vee$	$\dfrac{\phi}{\phi \vee \psi}\ \vee\mathrm{i}_1 \qquad \dfrac{\psi}{\phi \vee \psi}\ \vee\mathrm{i}_2$	$\dfrac{\phi \vee \psi \quad \boxed{\begin{array}{c}\phi\\ \vdots\\ \chi\end{array}} \quad \boxed{\begin{array}{c}\psi\\ \vdots\\ \chi\end{array}}}{\chi}\ \vee\mathrm{e}$
$\rightarrow$	$\dfrac{\boxed{\begin{array}{c}\phi\\ \vdots\\ \psi\end{array}}}{\phi \rightarrow \psi}\ \rightarrow\mathrm{i}$	$\dfrac{\phi \quad \phi \rightarrow \psi}{\psi}\ \rightarrow\mathrm{e}$
$\neg$	$\dfrac{\boxed{\begin{array}{c}\phi\\ \vdots\\ \bot\end{array}}}{\neg\phi}\ \neg\mathrm{i}$	$\dfrac{\phi \quad \neg\phi}{\bot}\ \neg\mathrm{e}$
$\bot$	(no introduction rule for $\bot$)	$\dfrac{\bot}{\phi}\ \bot\mathrm{e}$
$\neg\neg$		$\dfrac{\neg\neg\phi}{\phi}\ \neg\neg\mathrm{e}$

Some useful derived rules:

$$\dfrac{\phi \rightarrow \psi \quad \neg\psi}{\neg\phi}\ \mathrm{MT} \qquad \dfrac{\phi}{\neg\neg\phi}\ \neg\neg\mathrm{i}$$

$$\dfrac{\boxed{\begin{array}{c}\neg\phi\\ \vdots\\ \bot\end{array}}}{\phi}\ \mathrm{RAA} \qquad \dfrac{}{\phi \vee \neg\phi}\ \mathrm{LEM}$$

Fig. 1.2. Natural deduction rules for propositional logic.

Some Provable Equivalences

$p \vee q$	$\dashv\vdash$	$\neg(\neg p \wedge \neg q)$	$\neg(p \vee q)$	$\dashv\vdash$	$\neg p \wedge \neg q$
$p \rightarrow q$	$\dashv\vdash$	$\neg q \rightarrow \neg p$	$p \rightarrow q$	$\dashv\vdash$	$\neg p \vee q$
$p \wedge q \rightarrow p$	$\dashv\vdash$	$r \vee \neg r$	$p \wedge q \rightarrow r$	$\dashv\vdash$	$p \rightarrow (q \rightarrow r)$