CISC-365*
Test #4
March 26, 2019

Student Number (Required) ______________________

Name (Optional)________________________________

This is a closed book test. You may not refer to any resources.

This is a 50 minute test.

Please write your answers in ink. Pencil answers will be marked, **but will not be re-marked under any circumstances.**

The test will be marked out of 50.

Question 1	/15
Question 2	/35
TOTAL	/50

☐ By writing my initials in this box, I authorize Dr. Dawes to destroy this test paper if I have not picked it up by April 30, 2019.

Question 1 (15 marks)

Suppose we are solving a minimization problem using the Branch and Bound technique. Let P be a **partial solution**, and let l and u be the lower and upper bounds computed for P.

(a) [5 marks] Is it possible for some full solution that expands on P to have an actual cost x where $x >$ *Global Upper Bound U* ? Explain your answer.

Solution:

Yes. The Global Upper Bound U is an upper bound on the cost of the optimal solution, not on the cost of every solution. There can be non-optimal solutions that have extremely high costs.

Marking:

Yes, with a sound explanation	**5**
Yes, with weak explanation such as "The Global Upper Bound is used to eliminate partial solutions" which is True but not relevant here	**3**
Yes, with no explanation	**2**
No	**1**
No attempt	**0**

(b) [5 marks] Is it possible for a partial solution P' that expands on P to have bounds l' and u' such that $l' < l$? Explain your answer.

Solution:

No. The lower bound consists of Costs So Far and Guaranteed Future Costs. By definition, these costs cannot be avoided in any expansion of P. Thus l′ must be $\geq$ l

Marking:

No, with a sound explanation	**5**
No, with a weak explanation such as "The lower bound cannot decrease"	**3**
No, without explanation	**2**
Yes	**1**
No attempt	**0**

(c) [5 marks] Is it possible for a partial solution P' that expands on P to have bounds l' and u' such that $l' > u$? Explain your answer.

Solution:

Yes. The upper bound u is an upper bound on the cost of the best expansion of P, not an upper bound on the cost of every expansion of P. It is possible to generate an expansion of P that has cost > u, and it is possible that l' could equal that cost.

Marking:

As for Part (a)

Question 2 (35 marks):

You have accepted the job of coordinating a camping trip for a group of Canadian politicians. You are providing them with tents – each tent can accommodate exactly **four** campers. Your task is to divide the campers into groups of four. Fortunately the group contains 32 members so you know you will need exactly eight tents. The tents are numbered 1 to 8.

Unfortunately the members of the group don't like each other very much. You have been provided with a matrix A that records the levels of dislike between the individuals. A[i,j] = the level to which Person i dislikes Person j. Note that it is not necessarily true that A[i,j] = A[j,i]. The values in A are all in the range [1 .. 10]

The Unhappiness in a tent is the sum of the dislike values each person in a tent feels towards the other three people in the tent. The Group Unhappiness is the **maximum** of the Unhappiness of all the tents. For example if the Unhappiness values for the tents are $\{18, 24, 16, 19, 12, 17, 33, 27\}$ then the Group Unhappiness is 33.

In this question you will design a Branch and Bound algorithm to find the assignment of campers to tents that **minimizes** the Group Unhappiness.

For most parts of this question there are several possible answers. Answers that show deeper understanding of Branch and Bound methods will earn higher grades.

Note on marking this question: Some students may misunderstand the objective function – they may try to minimize the total Unhappiness instead of minimize the maximum Unhappiness in the set of tents (despite the given example!) Students who make this error should be penalized 7 points but their answers should be evaluated with respect to their misunderstanding of the question. For example if they use a greedy heuristic to try to minimize total Unhappiness instead of maximum Unhappiness, they should not lose marks twice for perpetuating their original error of understanding.

(a) [4 marks] Suppose there is a predefined function F(P,t) that returns the Unhappiness of tent t in solution P. Using this function, write code or pseudo-code to compute the Group Unhappiness for any solution P.

Solution:

```
GU = 0
for i= 1 to 8:
     if F(P,i) > GU:
          GU = F(P,i)
```

Marking:

For a solution that correctly chooses the maximum of the 8 F(P,i) values	**4**
For a solution that has the right idea but contains one or two errors	**2 or 3**
For trying	**1**
For no attempt	**0**

(b) [5 marks] Characterize your solution method as a sequence of decisions. Explain your reasoning.

Solution:

Number the Politicians from 1 to 32. The i^{th} decision is to choose a tent for Politician i. The choice is limited when some of the tents are full. For the last Politician in the list there is only 1 vacant spot so there is not really any decision to be made for the last Politician. If we use this method we can try to use a greedy heuristic that places each person where they are least unhappy.

OR

Number the Politicians from 1 to 32. The first 4 decisions are to choose the occupants of Tent 1, the next 4 decisions are to choose the occupants of Tent 2, etc. Once the occupants of the first 7 tents are chosen, the last four Politicians are assigned to Tent 8. If we use this method we can use a greedy heuristic to put together groups of 4 Politicians with low Unhappiness.

Marking:

For either of the answers above, with explanation	**5**
For any other feasible sequence of decisions, with explanation	**5**
For any feasible answer, without explanation	**3**
For an answer that shows understanding of the question but does not answer	**2**
For trying	**1**
For not trying	**0**

(c) [6 marks] How will you compute the initial value of the *Global Upper Bound* U? Explain your reasoning.

Solution:

Possible method 1: Assign the first four people in the list to Tent 1, the next four to Tent 2, etc

Possible method 2: Assign the first 8 Politicians to empty tents. For each remaining Politician, put them in the available tent in which they increase the Unhappiness the least. This tries to put each person where they will be least unhappy.

Possible method 3: Assign Politician 1 to Tent 1. Find the Politician j who has the lowest combined dislike with Politician 1 (ie find the j that minimizes A[1,j] + A[j,1]). Assign Politician j to Tent 1. Find the Politician k who has the lowest combined dislike with Politician 1 and Politician j, and assign k to Tent 1. Now find the Politician m who has the lowest combined dislike with Politicians 1, j and k. Assign Politician m to Tent 1. Now repeat for Tents 2 through 8, using the remaining Politicians. This tries to create tents that have low Unhappiness.

Marking:

For something like Method 2 or Method 3, or something similar that relates to the student's "sequence of decisions" answer – the important thing is to try to get an upper bound that isn't just randomly chosen. Explanation given.	**6**
For a good answer (as just explained) without explanation	**4**
For a weak answer such as Method 1, with explanation	**4**
For a weak answer without explanation	**3**
For an answer that shows understanding of the Global Upper Bound but does not produce one	**2**
For trying	**1**
For not trying	**0**

d) [6 marks] How will you compute the Cost So Far for partial solutions? Explain your reasoning.

Solution:

The Cost So Far is the Group Unhappiness calculated using the assignment of Politicians we have made so far. Explanation: this is simply the accurate measure of the cost of the decisions made up to this point.

Marking:

For something similar to the above, with explanation	**6**
For a good answer, without explanation	**4**
For an answer that shows understanding of the purpose of Cost So Far, without explaining how to find it	**3**
For an answer that shows limited understanding	**2**
For trying	**1**
For not trying	**0**

(e) [8 marks] How will you compute the Guaranteed Future Costs for partial solutions? Explain your reasoning.

Solution:

If there is a Politician not yet in a tent whose addition will raise the Group Unhappiness no matter which tent they occupy, then the minimum of those increases is a future cost – call this FC(P) where P is the politician. If this is true of several unassigned Politicians, we can take the largest of the FC(P) values as the GFC value.

If there is a tent with two, three or four vacancies, we can try all combinations of the remaining Politicians to fill that tent (this is $O(n^4)$ for the empty tent case). If EVERY combination raises the Group Unhappiness then the least amount by which it will go up is a guaranteed future cost. This is a lot of work for probably not much payoff, but it is mathematically valid.

Marking:

If the student is able to correctly identify any guaranteed future cost – ie a situation such as the ones described above that will unavoidably raise the Group Unhappiness – then they should get full marks (8/8)

For a solution that proposes a guaranteed future cost that is not valid (because it is possible but not guaranteed, or because it double counts in some way, etc.) but which accurately describes a situation that would increase the Group Unhappiness, give 7/8

For a solution that clearly demonstrates understanding of GFC as it applies to this problem, without specifying a computable cost, give 6/8

For a solution that demonstrates understanding of GFC in general, give 5/8

For a solution that shows weak understanding if GFC, give 3/8

For trying, give 1/8

(f) [6 marks] How will you compute the Feasible Future Costs for partial solutions? Explain your reasoning.

Solution:

I will apply exactly the same technique as described in Part (c) to extend P to a full solution. If the actual cost of this solution – calculated as in Part (a) – is < Global Upper Bound, then Global Upper Bound is reduced.

It may be worth trying several extensions of P in hope of finding one that gives a good reduction in U.

Marking:

Students may propose methods for computing FFC that are different from their methods for computing the initial value of the Global Upper Bound. As long as their FFC incorporates some attempt to find a good expansion (rather than just a randomly chosen one), give 6 marks

Basically apply the same marking scheme as for Part (c)